

Mark Scheme (Results)

January 2016

Pearson Edexcel International A Level in Mechanics 3 (WME03/01)



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#### **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

#### EDEXCEL IAL MATHEMATICS

#### **General Instructions for Marking**

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- **\*** The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 6. Ignore wrong working or incorrect statements following a correct answer.

### **General Principles for Mechanics Marking**

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- MARKS MUST BE ENTERED IN THE SAME ORDER AS THEY APPEAR ON THE MARK SCHEME.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
   M(A) Taking moments about A.
  - N2L Newton's Second Law (Equation of Motion)
  - NEL Newton's Experimental Law (Newton's Law of Impact)
  - HL Hooke's Law
  - SHM Simple harmonic motion
  - PCLM Principle of conservation of linear momentum
  - RHS, LHS Right hand side, left hand side.

# edexcel

Question Number	Scheme	Marks
1.	$R\sin 60^\circ = mg$	M1A1
	$R\cos 60^\circ = mr\omega^2$	M1A1
	$\tan 60^\circ = \frac{g}{r\omega^2}$	
	$\omega = \sqrt{\frac{g}{r\sqrt{3}}} = \sqrt{\frac{g\sqrt{3}}{3r}} *$	ddM1A1cso [6]
M1	Resolve vertically, allow with $60^{\circ}$ or $\theta$	
A1 M1	Correct equation, with 60° or $\theta$	
M1 A1	Equation of motion along radius, acceleration in either form, $60^{\circ}$ or $\theta$ Correct equation, with $60^{\circ}$ or $\theta$ acceleration to be $r\omega^2$ now	
ddM1	Eliminate <i>R</i> , substitute a numerical value for $\theta$ and solve to $\omega = \dots$ or $\omega^2 = \dots$ Depends on both previous M marks	
A1cso	Complete to the given answer	

Question Number	Scheme	Marks
2(a)	$a = \frac{dv}{dt} = 6 - 2t$ $v = 6t - t^{2} (+c)$ $t = 0, v = -8 \Longrightarrow c = -8$	
	$v = 6t - t^2 \ (+c)$	M1A1
	$t = 0, v = -8 \Longrightarrow c = -8$	
	$t = 0, v = -8 \implies c = -8$ $v = 6t - t^2 - 8$ $v = 6t - t^2 - 8 = 0$ (t - 2)(t - 4) = 0 t = 2, t = 4	A1 (3)
<b>(b)</b>	$v = 6t - t^2 - 8 = 0$	
	(t-2)(t-4)=0	
	t = 2, t = 4	M1
	$s = \int (6t - t^2 - 8) dt = 3t^2 - \frac{1}{3}t^3 - 8t  (+c)$	M1
	$\left[3t^{2} - \frac{1}{3}t^{3} - 8t\right]_{2}^{4} = 48 - \frac{64}{3} - 32 - \left(12 - \frac{8}{3} - 16\right) = 1\frac{1}{3}$	A1
	$\left[3t^{2} - \frac{1}{3}t^{3} - 8t\right]_{0}^{2} = 12 - \frac{8}{3} - 16(-0) = -6\frac{2}{3}$	A1
	Total distance = 8 m	A1ft (5) [8]
(a)		
M1	Using $a = \frac{dv}{dt}$ and attempting the integration Use of $a = v \frac{dv}{dx}$ scores M	40
A1 A1	Correct integration w/wo constant Find constant and correct statement for the velocity	
<b>(b)</b>		
M1 M1	Find the times when $P$ is at rest (Usual rules for factorising or formula) Integrate $v$ to obtain an expression for $s$ ( $c$ not needed)	
A1	Correct displacement for either time interval	
A1 A1ft	Correct displacement for second time interval Add 2 (positive) distances	
	NB: Using $v = 8$ Is NOT a misread.	

Question Number	Scheme	Marks
3	$R(\uparrow)  R\cos 20^\circ = F\cos 70^\circ + 800g$	M1A1A1
	NL2( $\rightarrow$ ) $R\cos 70^\circ + F\cos 20^\circ = 800\frac{v^2}{20}$	M1A1A1
	$F = \mu R = 0.5R$	
	$R\cos 20^\circ = 0.5R\cos 70^\circ + 800g$	
	$R = \frac{800g}{(\cos 20^\circ - 0.5\cos 70^\circ)}$	
	$v^{2} = \frac{1}{40} \frac{800g(\cos 70^{\circ} + 0.5\cos 20^{\circ})}{(\cos 20^{\circ} - 0.5\cos 70^{\circ})}  \text{OR} \ \frac{v^{2}}{20g} = \frac{(\cos 70^{\circ} + 0.5\cos 20^{\circ})}{(\cos 20^{\circ} - 0.5\cos 70^{\circ})}$	ddM1
	$v = 14.38 = 14.4$ or $14 \text{ m s}^{-1}$	dddM1A1cao [9]
M1 A1A1 M1 A1 A1 ddM1 dddM1 A1	Resolve vertically A1A1 if equation completely correct; A1A0 one error; A0A0 two or more errors Equation of motion along the radius, acceleration in either form LHS correct RHS correct inc acceleration in form $\frac{v^2}{r}$ <i>m</i> or 800 for these 2 marks Use $F = \mu R$ and eliminate <i>R</i> to obtain $v^2 = \dots$ Depends on both previous M marks Complete to a numerical value for $v^2$ or <i>v</i> All previous M marks needed. Correct value of <i>v</i> 2 or 3 sf only	

Question Number	Scheme	Marks	
4 (a)	$A \underbrace{4a}_{C} B$		
	$R(\uparrow)  2T \sin \theta = mg$ $\sin \theta = \frac{3}{5}  \text{or}  \cos \theta = \frac{4}{5}$ $T = \frac{5}{6}mg$	M1A1 B1	
	$T = \frac{\lambda \times 2a}{3a}$ $\lambda = \frac{3}{2} \times \frac{5}{6} mg = \frac{5}{4} mg *$	M1 A1 cso (5)	
	EPE at $D = \frac{\lambda a^2}{2 \times 3a}$ at $C = \frac{\lambda \times 4a^2}{2 \times 3a}$ (or equivalents with half strings)	B1 (either)	
	$\frac{1}{2}mv^2 + \frac{4\lambda a^2}{6a} = \frac{\lambda a^2}{6a} + mg \times \frac{3}{2}a$ $v^2 = \frac{7}{4}ag$	M1A1 dM1	
	$v = \frac{\sqrt{7ag}}{2}$	A1 (5) [10]	
(a) M1 A1 B1 M1 A1cso	Resolve vertically Must have 2 <i>T</i> . Correct equation A correct trig value for an angle - seen implicitly or used Hooke's law inc attempting the extension in terms of <i>a</i> Obtain the given value of $\lambda$ from correct working		
(b) B1	Obtain the correct EPE at either start or finish of the motion. May be those shown (full strings) or half of these (half strings) (May have already sub their $\lambda$ )		
M1 A1 M1 A1	An energy equation with the correct number of terms. EPE terms to be of the form $k \frac{\lambda x^2}{l}$ A fully correct equation Solve to $v^2 =$ A correct expression for $v$ Any equivalent form		

Question Number	Scheme	Marks	
5 (a)	$T = mg\sin\alpha = \frac{3}{5}mg$	B1	
	$T = \frac{\lambda \times \frac{1}{5}l}{l} = \frac{3}{5}mg$	M1	
	$\lambda = 3mg^{*}$	A1 cso (3)	
	$mg\sinlpha - T = m\ddot{x}$		
	$\frac{3}{5}mg - \frac{3mg\left(\frac{1}{5}l + x\right)}{l} = m\ddot{x}$	M1A1	
	$\ddot{x} = -\frac{3g}{l}x$	dM1	
	: SHM	A1 (4)	
(c)	$\left \ddot{x}\right _{\max} = a\omega^2 = \frac{1}{2}l \times \frac{3g}{l} = \frac{3g}{2}$	M1A1ft (2)	
( <b>d</b> )	Time from D to B: $\frac{1}{4}l = \frac{1}{2}l\sin\sqrt{\frac{3g}{l}}t_1$	M1	
	$t_1 = \frac{\pi}{6} \sqrt{\frac{l}{3g}}$		
	Time from <i>B</i> to nat. length: $\frac{1}{5}l = \frac{1}{2}l\sin\sqrt{\frac{3g}{l}}t_2$	M1	
	$t_2 = \sqrt{\frac{l}{3g}} \sin^{-1} \frac{2}{5}$		
	Total time $=\left(\frac{\pi}{6} + \sin^{-1}\frac{2}{5}\right)\sqrt{\frac{l}{3g}}, = 0.54\sqrt{\frac{l}{g}} \qquad k = 0.54$	ddM1,A1cao (4) [13]	
(a)B1 M1	$T = \frac{3}{5}mg$ shown explicitly or used Attempt Hooke's law		
A1cso	Obtain the given result		
(b)M1 A1	Equation of motion along the plane inc <i>T</i> in terms of <i>l</i> and <i>x</i> . Allow with according correct equation, acceleration can still be <i>a</i> . Can still have $\lambda$	eleration as a	
dM1	Re-arrange to the required form. Acceleration must be $\ddot{x}$ oe now		
A1	Correct result as shown or $\ddot{x} = -\frac{\lambda}{ml}x$ and SHM stated. If sub made for g, and	nswer must be 2	
	or 3 sf. But may be possible to award earlier.		
(c)M1	Use $ \ddot{x} _{\text{max}} = a\omega^2$		
A1ft (d)M1	Correct answer follow through their $\omega$ Must be positive 14.7 or 15 allowed Find time from D to B or from C to D		
M1	Find time from <i>B</i> to natural length or from <i>C</i> to natural length		
ddM1	Add or subtract (as appropriate) the two times obtained		
A1	Correct result for $k$ Must be 2 sf but need not be shown explicitly.		

Question Number			Scheme		Marks
6(a)	$(\pi\rho)\int_0^r xy^2\mathrm{d}x$				
	$(\pi\rho)\int_0^r xy^2 dx$ $= (\pi\rho)\int_0^r x(r^2 - (\pi\rho))\left[\frac{1}{2}x^2r^2\right]$ $= (\pi\rho)\frac{r^4}{4}$ $M\overline{x} = \pi\rho\int xy^2 dx$	$-x^2$ )dx			M1
	$= (\pi \rho) \left[ \frac{1}{2} x^2 r^2 \right]$	$-\frac{x^4}{4}\bigg]_0^r$			A1
	$=(\pi\rho)\frac{r^4}{4}$				A1
	$M\overline{x} = \pi\rho\int xy^2 \mathrm{d}x$	x			M1
	$\overline{x} = \frac{\pi \rho r^4}{4} \div \frac{2\pi \rho}{3}$	$\frac{2r^3}{8} = \frac{3}{8}r^*$			A1 (5)
(b)	Mass	т	M	( <i>m</i> + <i>M</i> )	
	Dist from O	$\frac{3}{8}r + 4r$	$\frac{3}{4} \times 4r$	$\overline{x}$	B1
	$\frac{35}{8}rm + 3rM =$	$(m+M)\overline{x}$			M1A1ft
	$\overline{x} = \frac{\left(35m + 24M\right)}{8\left(m + M\right)}$	$\frac{M}{r}$			A1 (4)
(c)	$\overline{x}\cos\theta \leqslant OA$				M1
	$\cos\theta = \frac{4r}{OA}$				B1
	$\cos \theta = \frac{4r}{OA}$ $\overline{x} \leqslant \frac{OA^2}{4r}$				A1
	$\overline{x} = \frac{\left(35m + 24M\right)}{8\left(m + M\right)}$	$\frac{M}{r}\right)r \leqslant \frac{17r^2}{4r}$			M1
	$35m+24M \leqslant 3$	34(m+M)			
	$M \geqslant \frac{m}{10} *$				A1 (5)
	10				[14]

Question Number	Scheme	Marks	
(a)			
M1	Using $\int xy^2 dx = \pi$ and/or $\rho$ may be missing limits need not be shown		
A1	Correct integration. $\pi, \rho$ and limits need not be shown		
A1	Use limits to obtain $\frac{r^4}{4}$		
M1	Use $M\overline{x} = \pi\rho \int xy^2 dx$ with their previous result. $\pi$ and $\rho$ must be seen in both sides or neither.		
A1cso	Obtain $\frac{3}{8}r$		
(b) B1 M1	Correct distances from <i>O</i> or centre of plane face (can both be positive). Construct a moments equation with their distances using their masses (which may be volumes)		
A1ft	Correct moments equation, follow through their distances (but not their masses). If working from centre of plane face one term must be negative now.		
A1 (c)	Correct result for $\overline{x}$ (any equivalent) (inc fractions within fractions)		
M1 B1	Attempting an inequality with $\overline{x}$ and <i>OA</i> (Sign either way round or =) A correct trig function connecting the angle used and <i>OA</i>		
A1	Obtain $\overline{x} \leq \frac{OA^2}{4r}$		
M1 A1cso	Use their expression for $\overline{x}$ in their inequality/equality. Obtain the given result.		
ALT for (c)			
	Use distance from centre of common face and tangents		
	<i>M</i> is min when $\frac{\overline{x} - 4r}{r} = \frac{r}{4r} (= \tan \theta)$		

Question Number	Scheme	Marks
7(a)	For complete circles there must be a speed at the top. (or equiv statement)	B1
	$\frac{1}{2}mu^2 > \frac{mgl}{5}$	M1A1
	$u > \sqrt{\frac{2gl}{5}} *$	A1 (4)
(b)	NL2 at bottom $T_{\text{max}} - mg = m\frac{v^2}{l}$	M1A1
	Energy to bottom $\frac{1}{2} \times mv^2 - \frac{1}{2} \times mu^2 = mgl(1 + \cos\alpha) = \frac{mg \times 9}{5}$	M1A1
	$T_{\max} = mg + \frac{m}{l} \left( \frac{18gl}{5} + u^2 \right)$	A1
	NL2 at top $T_{\min} + mg = m\frac{v^2}{l}$	M1A1
	$T_{\min} = \frac{m}{l} \left( u^2 - \frac{2}{5} gl \right) - mg$	M1A1
	$4\left(\frac{m}{l}\left(u^2 - \frac{2}{5}gl\right) - mg\right) = mg + \frac{m}{l}\left(\frac{18gl}{5} + u^2\right)$	dddM1
	$4\left(u^{2} - \frac{2}{5}gl - gl\right) = gl + \frac{18gl}{5} + u^{2}$	
	$4\left(u^{2} - \frac{2}{5}gl - gl\right) = gl + \frac{18gl}{5} + u^{2}$ $u = \sqrt{\frac{51gl}{15}} = \sqrt{\frac{17gl}{5}} *$	A1cso (11) [15]
(a) B1 M1 A1 A1	Statement (oe) shown seen in working. Use energy (and above statement) to obtain an inequality for $u^2$ Correct inequality Deduce the given result	
ALT B1,M1 A1A1	Energy equation including speed at top State $v^2 > 0$ and use in energy equation to obtain an inequality As above	

Question Number	Scheme	Marks
(b) M1 A1 M1 A1 A1 M1 A1 M1 A1 ddddM1 A1cso	Attempt NL2 at bottom. Acceleration in either form. Correct equation inc acceleration in $\frac{v^2}{r}$ form Energy equation from <i>A</i> to lowest point Correct energy equation Eliminate $v^2$ to obtain the max tension in terms of <i>m</i> , <i>g</i> , <i>l u</i> as shown oe Attempt NL2 at top. Acceleration in either form. Correct equation inc acceleration in $\frac{v^2}{r}$ form Use energy to top to obtain an expression for least tension in terms of <i>m</i> , <i>g</i> , <i>l</i> and Correct expression as shown oe Connect the two tensions using 4 on either side Obtain the given result.	и

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